The KPZ fixed point and Brownian motion have the same null sets

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Third Congress of Greek Mathematicians

Joint work with Sourav Sarkar

June 29 – July 4, 2026

Overview

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In 1986, Kardar-Parisi-Zhang (1986) predicted that many planar random growth processes with the above qualitative characteristics possess universal scaling behaviour, with local dynamics governed by the KPZ equation:

$$\partial_t h = \underbrace{(\partial_x h)^2}_{\text{lateral growth}} + \underbrace{\partial_x^2 h}_{\text{smoothing}} + \underbrace{\xi}_{\text{spacetime white noise}} \qquad \underbrace{h(t,x)}_{\text{height at time } t\text{, position } x}$$

Eden model

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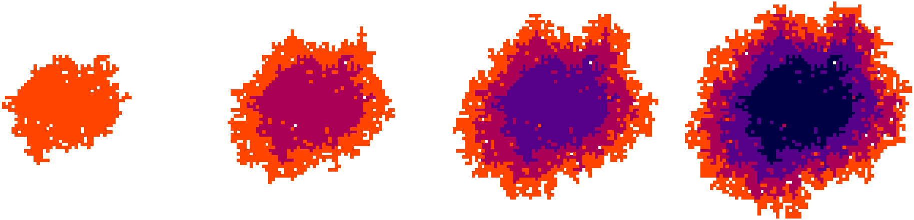

Figure: Simulation of Eden growth model at four different epochs, adding 800 new blocks per epoch.

Ballistic deposition

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Ballistic deposition model

Unit blocks fall independently and in parallel from the sky above each site of $\mathbb{Z}$ at rate 1 and **stick to the first edge against which they become adjacent to**.

Poisson process of falling blocks

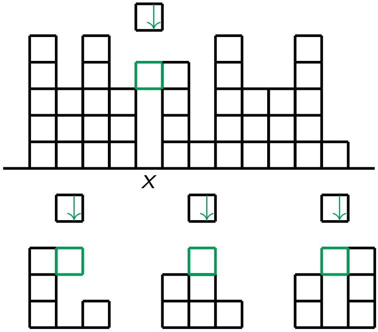
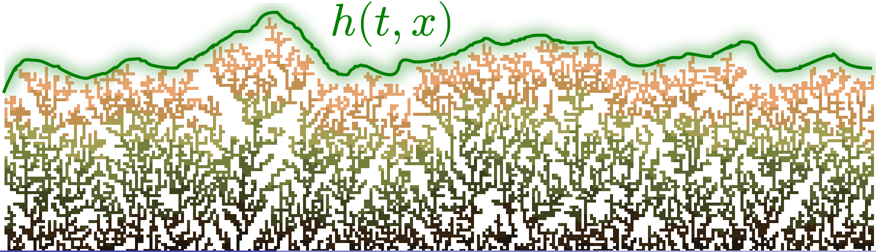


Figure: Ballistic deposition model. Growth occurs (here at site x) when blocks stick to first point of contact (denoted in green).

Ballistic deposition run for long time

Simulations suggest:



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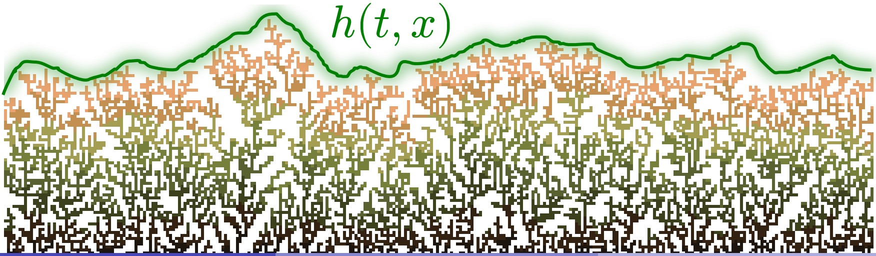

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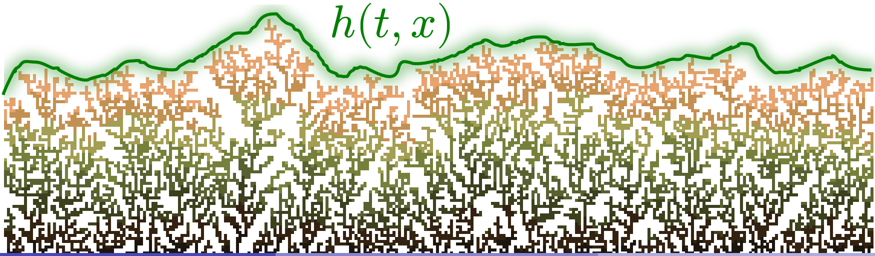

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$$\varepsilon^{1/2} h(\varepsilon^{-3/2} t, \varepsilon^{-1} x) - C_\varepsilon t$$

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- Fluctuation : Space : Time $= 1 : 2 : 3$

Overview

What is the KPZ universality class?

- All models in this class have a height function which has the "same asymptotic fluctuations" under the 1:2:3 scaling as the KPZ equation.

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(2) whose distribution is fixed under $1:2:3$ scaling, i.e., for all $\varepsilon > 0$, $\varepsilon^{1/2}\mathfrak{h}_{\varepsilon^{-3/2}t}(\varepsilon^{-1}x, \varepsilon^{-1/2}h_0(\varepsilon x)) \overset{dist.}{=} \mathfrak{h}_t(x, h_0)$.

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- Analogue of the Donsker's theorem in this universality class!
- *A model is in the KPZ universality class, if starting from* **any height**, *its height under* $1:2:3$ *scaling converges to the KPZ fixed point.*

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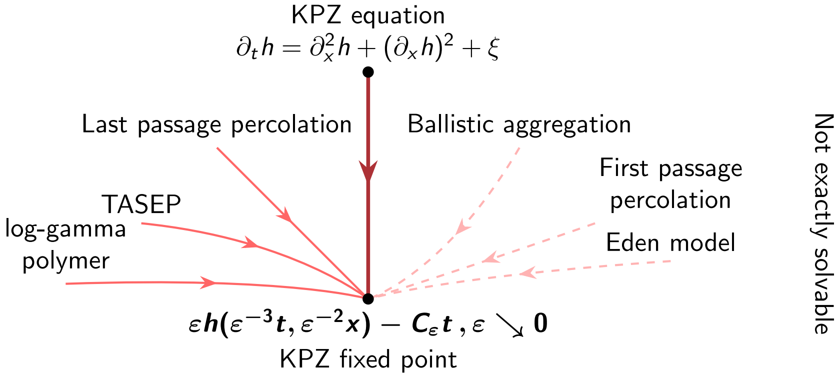

KPZ fixed point as a variational formula

- The KPZ fixed point at time 1 can be written in terms of a random continuous environment, the Airy sheet $\mathcal{S} : \mathbb{R}^2 \to \mathbb{R}$, constructed in Dauvergne-Ortmann-Virág (2018) and the initial condition $h_0 : \mathbb{R} \mapsto \mathbb{R} \cup \{-\infty\}$ as

$$\mathfrak{h}_1(y) = \sup_{x \in \mathbb{R}} \Big(h_0(x) + \mathcal{S}(x, y) \Big), \qquad y \in \mathbb{R}.$$

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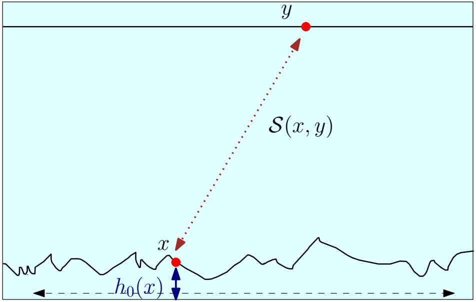

Airy sheet profile

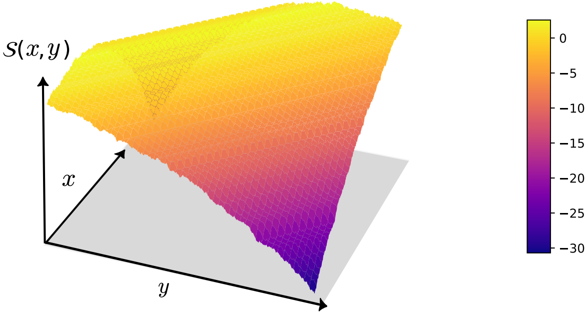

Figure: Simulation of the Airy sheet on $[-3, 3] \times [-3, 3]$ using approximation by Brownian last passage values. Note the parabolic curvature.

KPZ fixed point for special initial conditions

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- What can we say about the KPZ fixed point started from arbitrary initial condition?
- Does it still look "locally like a Brownian motion"?

Which is which?

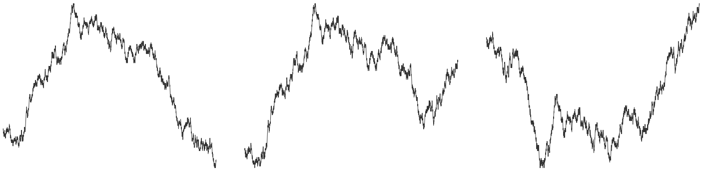

Figure: Which is which? The KPZ fixed point started from the narrow wedge at zero, flat initial data ($h_0 \equiv 0$, also known as the Airy$_1$ process), and a Brownian motion.

Overview

Definitions: local Brownianness

Local Brownian limit

F has local Brownian limit if the scaled process $\varepsilon^{-1/2}(F(\cdot + \varepsilon \cdot') - F(\cdot)) \xrightarrow{d} B(\cdot')$ as $\varepsilon \to 0$, where B is a Brownian motion.

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Brownian on compact

F is called Brownian on compact if for all $y_1 < y_2$, the law of $F(\cdot) - F(y_1)$ on $[y_1, y_2]$ is absolutely continuous wrt that of a Brownian motion (with diffusion parameter 2) starting from $(y_1, 0)$ on $[y_1, y_2]$.

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- Brownian on compact $\implies$ local Brownian limit.

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- Calvert-Hegde-H (2019): For general initial conditions – Brownian patchwork quilt. Question left open: if it can be reduced to a single patch?
- Sarkar-Virág (2021): For all initial conditions, the KPZ fixed point at any time t is Brownian on compacts.

Main result

Mutual absolute continuity, (T-Sarkar 26)

Fix an interval $[a, b]$ and $t > 0$. The law ν of the KPZ fixed point $\mathfrak{h}_t(\cdot, h_0) - \mathfrak{h}_t(a, h_0)$ started from arbitrary initial data h_0, is mutually absolutely continuous against the law of rate two Brownian motion on $[a, b]$.

Overview

Proof strategy

- The Airy sheet can be coupled to a random environment comprising of countably many non-intersecting random continuous functions on the real line, the *Airy line ensemble* $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \ldots)$, where $\mathcal{A}_1$ is the parabolic Airy_2 process.
- Using this coupling, we can now express the KPZ fixed point at time 1, as a functional over the Airy line ensemble, namely,

$$\mathfrak{h}_1(y, h_0) = F(\mathcal{A}_1|_{[0,y_0]}, \mathcal{A}|_{\{1,\ldots\}\times(-\infty,0]\cup[y_0,\infty)}, \mathcal{A}|_{\{2,\ldots\}\times[0,y_0]}, h_0)(y) \quad \text{for } y \in [0, y_0]. \tag{1}$$

- The structure of (1) is such that the increments of the KPZ fixed point in some $[a, b] \subset (0, y_0)$ become just the increments of $\mathcal{A}_1$, provided $\mathcal{A}_1$ avoids a certain 'barrier' on $[0, y_0]$ depending on the data $\mathcal{A}|_{\{1,\ldots\}\times(-\infty,0]\cup[y_0,\infty)}$ and $\mathcal{A}|_{\{2,\ldots\}\times[0,y_0]}$.
- The key tool that makes this possible is the *Brownian Gibbs property* for the Airy line ensemble.

Brownian Gibbs property

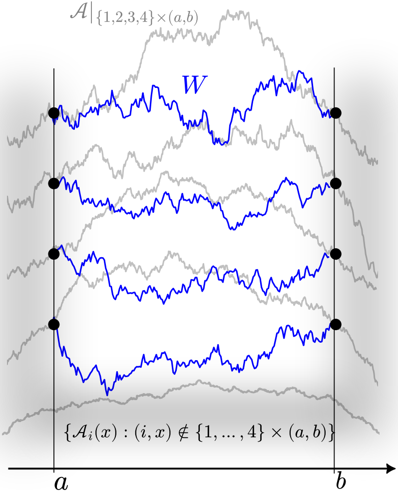
The **Brownian Gibbs resampling property** of the Airy Line ensemble [Corwin and Hammond, 2014], states that for $a < b$, $k \in \mathbb{N}$, the law of the Airy line ensemble in the region

$$\{1, 2, \cdots, k\} \times (a, b),$$

conditionally on

$$\{\mathcal{A}_i(x) : (i, x) \notin [\![1, k]\!] \times (a, b)\},$$

( ) is given by k independent Brownian bridges W conditioned to not intersect and stay above $\mathcal{A}_{k+1}$ on (a, b).



Brownian sandwitch

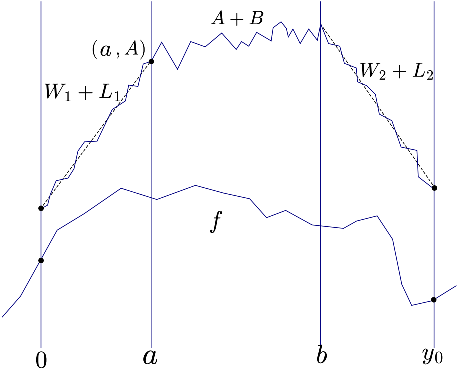


Figure: Illustration of a 'Brownian sandwitch' with W_1, W_2 Brownian bridges starting and ending at zero, affine functions L_1, L_2 (L_1 depends on A and L_2 on A, B) and a Brownian motion $A+B$ for some random variable A with support unbounded from above. Here, B, W_1, W_2, A and the 'lower barrier' f are mutually independent. We want to show that conditionally on B, the entire top line $W_1+L_1, A+B, W_2+L_2$, avoids f with positive probability. This is achieved taking $A \gg 1$, $||W_1||_\infty, ||W_2||_\infty \ll 1$ (depending on the sample paths of B).

Overview

Applications: Record times

We obtain as a direct consequence of Theorem 18 a result for the times the KPZ fixed point attains its running maximum (relative to some starting point).
Define the set of *record times* of the KPZ fixed point $\mathfrak{h}_t(\cdot, h_0)$

$$\mathscr{R}(a, t, h_0) \stackrel{\text{def}}{=} \{y \geq a : \mathfrak{h}_t(y, h_0) = \max_{a \leq s \leq y} \mathfrak{h}_t(s, h_0)\}, \quad a \in \mathbb{R}.$$

Corollary

In any interval $[b, c]$, $a < b < c$, the set of record times $\mathscr{R}(a, t, h_0) \cap [b, c]$ is non-empty with probability strictly between zero and one.

Applications: Hitting probabilities and capacity

Let E, F be compact subsets of $\mathbb{R}$. Let $\mathfrak{h}$ denote the KPZ fixed point (suppressing time dependence and the finitary initial data). We are interested in giving necessary and sufficient conditions for the positivity of probabilities of the form

$$\mathbb{P}(\mathfrak{h}(E) \cap F \neq \emptyset) = \mathbb{P}(\mathrm{Gr}(\mathfrak{h}) \text{ hits } E \times F) > 0\,, \tag{2}$$

where the graph of the KPZ fixed point is denoted by

$$\mathrm{Gr}(\mathfrak{h}) \stackrel{\text{def}}{=} \{(t, \mathfrak{h}(t)) : t \in \mathbb{R}\} \subseteq \mathbb{R}^2.$$

In other words, we are interested in the probability the KPZ fixed point on some compact E hits another compact F, (Figure 6 below).

Applications: Hitting probabilities and capacity

$$E \times F \cap \mathrm{Gr}(\mathfrak{h}) = \blacksquare \cap \diagup\!\diagdown\!\diagup$$

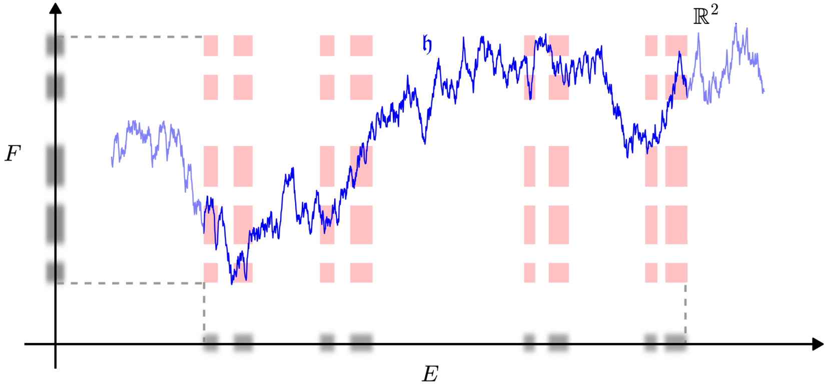


Figure: Illustration of the graph of the rescaled increments of the KPZ fixed point in blue and the set $E \times F$ in red. The probability $\mathfrak{h}(E) \cap F \neq \emptyset$ if and only if the graph of the KPZ fixed point, $\mathrm{Gr}(\mathfrak{h})$ hits the red region with positive probability.

Hitting probabilities and capacity

Corollary

- *If $F \subset \mathbb{R}$ is compact and $|F| > 0$, then*

$$\left\|\dim_{\mathscr{H}}\left(\mathfrak{h}(E) \cap F\right)\right\|_{L^\infty(\mathbb{P})} = 2\dim_{\mathscr{H}}(E) \wedge 1. \tag{3}$$

  If in addition, $\dim_{\mathscr{H}}(E) > 1/2$, then $\mathbb{P}(|\mathfrak{h}(E) \cap F| > 0) > 0$.

- *Suppose $F \subset \mathbb{R}$ is compact and has Lebesgue measure 0. Then $\mathbb{P}(\mathfrak{h}(E) \cap F \neq \emptyset) > 0$ if and only if $C_0(E \times F) > 0$. Moreover,*

$$\left\|\dim_{\mathscr{H}}\left(\mathfrak{h}(E) \cap F\right)\right\|_{L^\infty(\mathbb{P})} = \sup\left\{\gamma > 0 : C_\gamma(E \times F) > 0\right\}, \tag{4}$$

  where $C_\gamma(E \times F)$ is the γ-thermal capacity of $E \times F$ defined by

$$C_\gamma(E \times F) = \frac{1}{\inf\{\mathscr{E}_\gamma(\mu) : \mu \in \mathscr{P}(E \times F)\}}, \tag{5}$$

Thermal capacity

Definition

Let $E \subset \mathbb{R}$ and $F \subset \mathbb{R}$ be compact sets. For $\gamma \geq 0$, the γ-thermal capacity of the set $E \times F$ is defined by

$$C_\gamma(E \times F) = \frac{1}{\inf\{\mathscr{E}_\gamma(\mu) : \mu \in \mathscr{P}(E \times F)\}}, \tag{6}$$

where $\mathscr{P}(E \times F)$ denotes the set of all Borel probability measures μ on $E \times F$ such that $\mu(\{t\} \times F) = 0$ for all $t \in E$ and $\mathscr{E}_\gamma(\mu)$ is the γ-thermal energy of μ defined by

$$\mathscr{E}_\gamma(\mu) \stackrel{\text{def}}{=} \int_{E\times F} \int_{E\times F} \frac{\mathrm{e}^{-|x-y|^2/(4|t-s|)}}{|t-s|^{1/2} \cdot |x-y|^\gamma} \mu(\mathrm{d}s\,\mathrm{d}x)\mu(\mathrm{d}t\,\mathrm{d}y)\,. \tag{7}$$

Overview

Future directions!

- Can we distinguish the KPZ fixed points started from different initial data? (Theorem 18 shows this has to necessarily be global)

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- Conjecture in C-H-H (2019): can we prove the Radon-Nikodym derivative is in L^p for all $p > 1$?

Future directions!

- Conjecture in C-H-H (2019): can we prove the Radon-Nikodym derivative is in L^p for all $p > 1$?

Quantitative Brownian regularity, (T-Sarkar 25)

The law of the KPZ fixed point started from arbitrary (finitary) initial data on some fixed interval, ν, exhibits quantitative Brownian regularity of the form

$$\nu(A) \leq c \exp\left(-d \log^r \left(1/\mu(A)\right)\right) ,$$

for some positive constants $c, d > 0$, $r \in (0, 1)$ and all events A, where μ denotes the Wiener measure.

Ευχαριστώ για την προσοχή σας!

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